Some sample Quantum Mechanics problems from Atkins

9. Size of quantum $\Rightarrow$ $E = h\nu$ $\quad\nu = c/\lambda$
$h = 6.627 \times 10^{-34}$ J • S $\quad = 1/\tau$

10. Watts = J/second $\quad$ photon/second = $h\nu$/sec
10^7 photon/10 μ second = $10^7/10 \bullet 10^{-6}$ = 10^{12} photon/second

11. de Broglie problem $\quad p = h/\lambda = \nu$
recall J = kg m^2/s^2
$1g \times 1$ m $s^{-2} = 10^{-3}$ k_g m s^{-1}
$= 10^{-3}$ J • S • m^{-1}
$\lambda = 6.6 \times 10^{-34}$ J • S/10^{-3} J • S m^{-1} = $6.6 \times$ 10-31 m

14. Photon pressure momentum problem $\rightarrow$ photon absorbed
can view as energy $\quad h\nu \rightarrow 1/2\ m\ \nu^2$ (heating / lack $\rightarrow$)
or momentum $\quad$ **p** = $h/\lambda = m\nu$ (photon **k** property)
$F = m_a = dp/dt \Rightarrow (h/\lambda)$ (note photon)
(Problem 14 incomplete) $\quad$ – assume 10^{15} photon/second laser
– assume all hit sail
$(6.6 \times 10^{-34}$ J • S/650×10^{-7} m) $(10^{15}\ s^{-1}) \sim 10^{-8}$ N
$\rightarrow$ if even on sail can calculate
Pressure = F/A
$A = 1\ km^2 = 10^6\ m^2$ $\quad p \sim 10^{-14}\ N/m^2$
1 kg space craft $\rightarrow$ what is sail? At km^2!!
$\upsilon = a \bullet t = (F/m)\ t$

15. Uncertainty $\quad \Delta p\Delta x > \hbar/2 \quad \Delta p = 10^{-4} \bullet p$; $\Delta x \geq (\Delta p)^{-1}\ (\hbar/2) \sim$

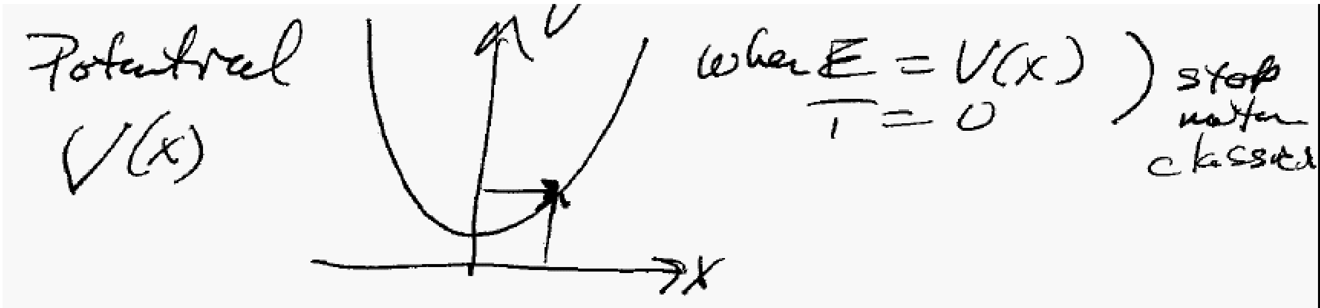
Topics of use in Quantum Mechanics:

Electrostatistics
Interaction electron-proton $\quad -e^2/r \quad r = a_0 = 0.53$ Å
problem here is units $\quad e = 1.6 \times 10^{-19}$ C
in MKS $\quad U = q_1q_2/4\pi\varepsilon_0 r \quad$ r = 0.53 × 10-10 m
q – C , r – m
$\varepsilon = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$(1.6 \times 10^{-19}\ C)^2/4\pi\ (8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2})\ (0.53 \times 10^{-10}\ m)$

$\sim 3 \times 10^{-38}/(4\pi \times 0.5)$ ($\quad$) 10^{-22} $\sim 0.5 \times 10^{-15}$ J
~6 $\quad$ ~9
macro: $U = 1389\ q_1q_2/r$ (kJ/mol)

Classical box – spacing order kT
He in 1μ value 25° C and 1° K
$E = \hbar^2\pi^2n^2/2ma^2 = \hbar^2n^2/8ma^2 \quad$ 1 – 0 $\quad \Delta E_{3D} = \Delta E_{1D} \quad$ n = 1 $\rightarrow$ n = 2



Acceptable w/f $\rightarrow$ integrable / finite

e^{ax}	$(-\infty < x < \infty)$	– nope – blows up
$e^{im\phi}$	$(0 < \phi < 2p)$	– yes m integer (continuous) – no – non continuous
x	$(0 < x < 10)$	– no – discontinuity (tricky, needs be zero)
$\lvert x - 10 \rvert$	$(-10 < x < 10)$	– okay, but not good (1st derivative not continuous at x = 0)
sin ax	$(-\pi/a < x < \pi/a)$	– yes
cos ax	– same	

Operator Linear / Hermitian $\int \psi_J^* \alpha \psi_i \, d\tau = \int \psi_i \alpha^* \psi_J \, d\tau$

Note $\alpha \psi_i = \alpha_i \psi_i$

$\alpha^* \psi_i^* = \alpha_i^* \psi_i^*$

$\int \psi_i^* \alpha \psi_i = \int \psi_i \alpha^* \psi_i^*$

$a_i \int \psi_i^* \psi_0 = a^* \int \psi_i \psi^*$

$a_i = a^*_i$ $\qquad$ a_i – real

Derive 3D KE $\quad$ KE 1D $= -\hbar^2/2m \, d^2/dx^2$

$= p^2/2m = \hbar^2/2m \, (d/dx \bullet d/dx)$

3D momentum

$(\hbar/i)(d/dx \, \hat{i} + d/dy \, \hat{j} + d/dz \, \hat{k}) = \hbar/i \, \nabla$

$p^2/2m = -\hbar^2/2m \, (\nabla \bullet \nabla) = -\hbar^2/2m \, (d^2/dx^2 + d^2/dy^2 + d^2/dz^2)$

Any momentum: $\mathbf{L} = \mathbf{n} \times \mathbf{p}$ review cross product

$$\mathbf{r} \times \mathbf{p} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix} = \begin{matrix} \mathbf{i}\,(y\,p_z - p_y\,z) \; + \\ \mathbf{j}\,(z\,p_x - p_z\,x) \; + \\ \mathbf{k}\,(x\,p_y - p_x\,y) \end{matrix}$$

$= L_x \mathbf{i} + L_y \mathbf{j} + L_z \mathbf{k}$

keep order in case commutivity is important

$L_x = y\, p_z - p_y\, z = y\, (\hbar/i\ \ \partial/\partial z) - (\hbar/i\ \ \partial/\partial y)\, z$

$= (\hbar/i)\, (y\, \partial/\partial z - z\, \partial/\partial y)$

$\partial/\partial y$ rotation z

Free particle:
$H\psi = E\psi = (T + U)\, \psi$
$= (-\hbar/2m\, \partial^2/\partial x^2)\psi = E\psi$

Solution: e^{ikx} $(-\hbar^2/2m)\, (ik)^2\, \psi = E\psi$
$E = \hbar^2 k^2/2m$

all energies are possible no restrictions k
→ propagation vector

$\hbar K = \sqrt{E/2m} = p = \hbar/\lambda$
$k = 2\pi/\lambda$

Constrain particle
classically bounce back from wall
Quantum Mechanics – slight penetration
due to wave continuous at x = 0