Problem
3.

3A. Prove that $n(x,t) = \frac{1}{\sqrt{4\pi D t}} e^{-x^2/4Dt}$

Satisfies $\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}$

Write $n = a t^{-1/2} e^{-bx^2/t}$ where $b = \frac{1}{4D}$

$$\frac{\partial n}{\partial t} = \left(-\frac{1}{2t} + \frac{bx^2}{t^2}\right) n$$

$$\frac{\partial n}{\partial x} = -\frac{2bx}{t} n$$

$$\frac{\partial^2 n}{\partial x^2} = -\frac{2b}{t} n + \frac{4b^2x^2}{t^2} n$$

$$= -\frac{n}{4Dt} + \frac{4bx^2}{4Dt^2} n = \frac{1}{D} \frac{\partial n}{\partial t}$$

B. $\int_{-\infty}^{\infty} e^{-x^2/4Dt} dx = \sqrt{4Dt} \int_{-\infty}^{\infty} e^{-z^2} dx = \sqrt{4\pi Dt}$

$\therefore \int_{-\infty}^{\infty} n(x)\, dx = 1$

C. $\langle x^2 \rangle = \frac{1}{\sqrt{4\pi Dt}} \int_{-\infty}^{\infty} x^2 e^{-x^2/4Dt} dx = \frac{(4Dt)^{3/2}}{\sqrt{4\pi Dt}} \int_{-\infty}^{\infty} z^2 e^{-z^2} dz$

$$= \frac{4Dt}{\sqrt{\pi}} \frac{\sqrt{\pi}}{2} = 2Dt$$