Note on the normalization of the normal mode eigenvectors.

The normalization constant is a little tricky, owing to the fact that we don't have the traditional secular equation. If the problem were

$$\vec{\vec{V}}\vec{a} = l\,\vec{a}\ ,$$

the normalization would be simply

$$\vec{a}\cdot\vec{a} = 1$$

or

$$\sum_i a_i^2 = 1$$

But here we have the more complicated case of

$$\vec{\vec{V}}\vec{a} = l\,\vec{\vec{T}}\vec{a}$$

Goldstein, in the text *Classical Mechanics*, chapter 10, shows that the generalized normalization is

$$\vec{a}\cdot\vec{\vec{T}}\cdot\vec{a} = 1$$

or

$$\sum_{i,j} T_{ij} a_{ik} a_{jk} = 1$$

For example, the first eigenvector becomes

$$\frac{1}{\sqrt{2m+M}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

With

$$\vec{\vec{T}} = \begin{pmatrix} m & 0 & 0\\ 0 & M & 0\\ 0 & 0 & m\end{pmatrix}$$

you can readily see that the normalization criterion is satisfied.