

Analytical Chemistry Cumulative Examination  
Professor M. Trenary  
March 4, 2010

1. (30 pts) An important aspect of analytical chemistry concerns the propagation of errors in measurements. A systematic approach to this problem involves a straightforward application of calculus. If  $u(x,y,z)$  is a function of  $x$ ,  $y$ , and  $z$ , then the spread in  $u$  determined by a given spread in  $x$ ,  $y$ , and  $z$  is given by

$$\Delta u = \sqrt{\left(\frac{\partial u}{\partial x}\right)_{y,z}^2 (\Delta x)^2 + \left(\frac{\partial u}{\partial y}\right)_{x,z}^2 (\Delta y)^2 + \left(\frac{\partial u}{\partial z}\right)_{x,y}^2 (\Delta z)^2}$$

a) (15 pts) Suppose a physical quantity,  $u$ , is related to measurable quantities  $x$ ,  $y$ , and  $z$  by the equation

$$u = x+y+z$$

and that the measured values of  $x$ ,  $y$  and  $z$  along with their uncertainties are  $3.25 \pm 0.03$ ,  $10.60 \pm 0.02$ , and  $7.60 \pm 0.08$ , respectively. Give the value of  $u$  along with its uncertainty.

b) (15 pts) Suppose a researcher wanted to estimate the radius of a Xe atom from the crystal structure of solid xenon determined at 58 K, which yields an average volume per atom of  $80.36 \pm 0.05 \text{ \AA}^3$ . The volume of a sphere is given by  $V = (4/3)\pi r^3$ . If the relative uncertainty in the volume,  $\Delta V/V = 0.05/80.36 = 6 \times 10^{-4}$ , what is the relative uncertainty of the radius,  $\Delta r/r$ ?

2. (40 pts) In using analytical instruments, a basic understanding of electricity is important.

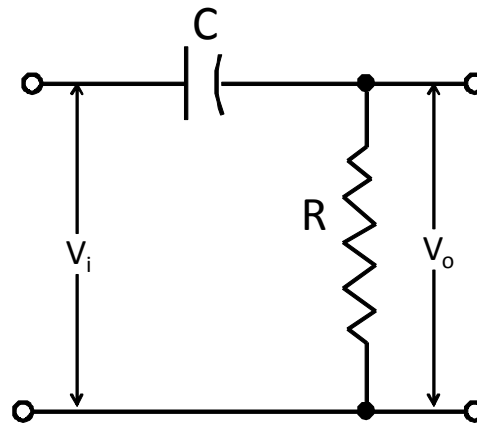
a) (10 pts) State Ohm's law. Define all quantities.

b) (10 pts) Give an expression for the power dissipated in a circuit with a current  $I$  and a voltage  $V$ .

c) (10 pts) What is the equivalent resistance,  $R_{eq}$ , of three resistors, of resistances  $R_1$ ,  $R_2$ , and  $R_3$ , arranged in series.

d) (10 pts) What is the equivalent resistance,  $R_{eq}$ , of three resistors, of resistances  $R_1$ ,  $R_2$ , and  $R_3$ , arranged in parallel.

3. (30 pts). High pass electronic filters are very common in analytical instruments that involve time dependent voltages. The figure below shows an equivalent circuit for a high pass filter.



a) (15 pts) For ac circuits, the effective resistance governing the relationship between peak values of the current and the voltage is referred to as the impedance,  $Z$ , which for this circuit is given by

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

where  $R$  is the resistance,  $C$  is the capacitance, and  $\omega$  is the frequency of the input voltage,  $V_i$ . Given that the peak values of the voltages and the currents are directly proportional to their root-mean-square values (rms), find the ratio of the rms values of the output voltage to the rms value of the input voltage, i.e., find  $(V_o/V_i)$ . Assume that the current going through the capacitor continues through the resistor.

b) (15 pts) Give an expression for the frequency at which the output voltage is half of the input voltage.